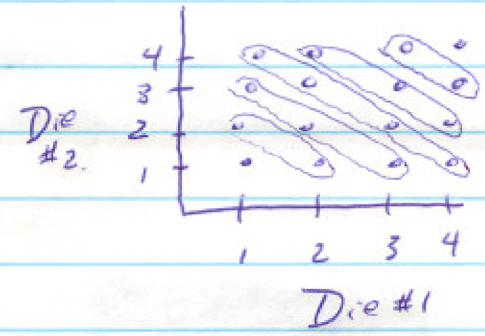
Elements of Statistics Final Exam - Fall 2004

Solutions

1. a.



$S = \{(1,1), (1,2), (1,3), (1,4),$ or $(2,1), (2,2), (2,3), (2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2), (4,3), (4,4)\}$

b. $S_2 = \{3, 4, 5, 6, 7, 8, 9\}$

c.

X = Character's Intelligence	3	4	5	6	7	8	9
Probability	1/16	2/16 = 1/8	3/16	4/16 = 1/4	3/16	2/16 = 1/8	1/16
or	.0625	.125	.1875	.25	.1875	.125	.0625

d. Mean Character's Intelligence $= \mu = 3\left(\frac{1}{16}\right) + 4\left(\frac{2}{16}\right) + 5\left(\frac{3}{16}\right) + 6\left(\frac{4}{16}\right) + 7\left(\frac{3}{16}\right) + 8\left(\frac{2}{16}\right) + 9\left(\frac{1}{16}\right)$

$\mu = 6$

e. Std dev of Character's Intelligence:

Variance $= \sigma^2 = (3-6)^2 \times \frac{1}{16} + (4-6)^2\left(\frac{2}{16}\right) + (5-6)^2\left(\frac{3}{16}\right) + (6-6)^2\left(\frac{4}{16}\right) + (7-6)^2\left(\frac{3}{16}\right) + (8-6)^2\left(\frac{2}{16}\right) + (9-6)^2\left(\frac{1}{16}\right)$

$\sigma^2 = 2.5$

$\sigma = \sqrt{2.5} = 1.58114$

2. $X \sim B(n=6, p=.7)$

a. $P(X=6) = .117649$

b. $P(X=4) = .324135$

c. $P(X \geq 4) = 1 - P(X<4) = 1 - P(X \leq 3) = 1 - .25569 = .74431$

d. $P(X \leq 4) = .579825$

Elements of Statistics Final Exam - Fall 2004

#3. a. A scatterplot shows a moderate/weak negative association between sweetness and pectin. That is, as the amount of pectin is increased, the sweetness level tends to decrease.
$r = -.478$.

b. $\widehat{Sweetness} = 6.2521 - .0023106$ Pectin

or $\widehat{sweetness} = \underset{\uparrow}{6.25} - .00231$ Pectin ← As pectin increases by 1 ppm the sweetness level will decrease by .00231

sweetness level when no pectin is present

c. Fit $= 6.2521 - .0023106(224) = 5.734487$

Resid = obs - predicted $= 5.8 - 5.734487 = .065512$

d. The normal probability plot shows roughly linear trend, with one exception in the lower tail. It is reasonable to assume that the residuals are normally distributed.

e. $H_0: \beta_1 = 0$ vs $H_1: \beta_1 \neq 0$

$t = \dfrac{-.0023106}{.0009049} = -2.55$ P-value $= .018$

Since $.018 < .05$, we reject H_0 and conclude that there is a significant linear association between sweetness and pectin.

f. 90% C.I. for β_1 is: $b_1 \pm t^*_{22} s_{b_1}$

$$-.0023106 \pm (1.71714)(.0009049)$$

$$(-.0039, -.0008)$$

or $(-.00386, -.000756)$

We are 90% confident that slope of the true regression line for predicting sweetness from pectin is between $-.0039$ and $-.0008$.
Notice that zero is NOT in this interval.

Elements of Statistics Final Exam - Fall 2004

4 a. $\hat{p}_{HF|Abstain} = .1629$ or 16.29%

b. $\hat{p}_{HF|<7\,drinks} = .1523$ or 15.23%

c. $\hat{p}_{HF|>7\,drinks} = .0903$ or 9.03%

d. These proportions are not the same so it does appear that congestive heart failure depends on alcohol consumption for AMI patients. More specifically, the more alcohol the patients consumed, the lower the rate of congestive heart failure.

e. H_0: Congestive Heart Failure and Alcohol Consumption are Independent
H_1: Congestive Heart Failure and Alcohol Consumption are Dependent

$\chi^2 = 10.197$, d.f. = 2, P-value = .006

Since $.006 < .01$, we reject H_0 and conclude that the rate of congestive heart failure depends on the level of alcohol consumption.

5. a. Matched pairs -

b. Let diff = Pattern2 - Pattern1

90% CI for μ_{diff} is $\bar{x}_{diff} \pm t^*_{14} \frac{s_{diff}}{\sqrt{n}}$

$$(-.312470, -.166196)$$

We are 90% confident that the mean difference (2 - 1) is between $-.312470$ and $-.166196$. Since zero is not in this interval we have significant evidence (at the $\alpha = .1$ level) that there is a difference in the pupil readings for the two patterns.

Elements of Statistics Final Exam - Fall 2004

#5 c. Diff = Pattern 2 - Pattern 1

$H_0: \mu_{diff} = 0$ vs $H_1: \mu_{diff} \neq 0$

$t = -5.76$ P-value = 0.000

Since $0.000 < .05$, we reject H_0 and conclude that there is a significant difference in the pupil readings for the two patterns. This conclusion is consistent with the interpretation of the C.I. (statistically significant evidence of a difference in the pupil readings).

d. We must assume that the differences are normally distributed and that the readings from ~~consume~~ different consumers are independent.

6. $H_0: P_{sun\text{-}shaded} = P_{unshaded}$

$H_1: P_{sun\text{-}shaded} > P_{unshaded}$

Wilson: $\tilde{p}_{ss} = \frac{35}{72}$ $\hat{p}_{ss} = \frac{34}{70}$

$\tilde{p}_{u} = \frac{32}{82}$ $\hat{p}_{u} = \frac{31}{80}$

$z = 1.20$

P-value = .116 (2-sided .231)

Traditional: (Not full credit)

$z = 1.21$

P-value = .113 $\binom{\text{2-sided}}{.226}$ don't pool .224

$\chi^2 = 1.467$ ($p = .226$)

Since $.116 > .01$ we cannot reject H_0. There does not appear to be a significant difference between the hatching rates for sun-shaded eggs and un-shaded eggs.

#7. a. The bone ratios (length-to-width) are roughly symmetric with a center around 9. (There is a slight gap at 7.2.)

b. center: mean = 9.258, median = 9.2, trimmed mean = 9.268

spread: $s = 1.204$, IQR = 9.935 - 8.615 = 1.32

Range = 5.77

Elements of Statistics - Fall 2004

#7. c. $H_0: \mu = 8.5$ vs $H_1: \mu \neq 8.5$

$t = 4.03$, P-value $= 0.000$

Since $0.000 < .01$, we reject H_0 and conclude that the mean length-to-width ratio differs from 8.5. These bones do not appear to belong to species A.

d. Use the transformation - $\dfrac{\text{ratio} - \text{mean}}{\text{std dev}} = \dfrac{\text{ratio} - 9.25756}{1.20357}$

OR $\quad Y = a + b(\text{ratio}) = -\dfrac{9.25756}{1.20357} + \dfrac{1}{1.20357}(\text{ratio})$

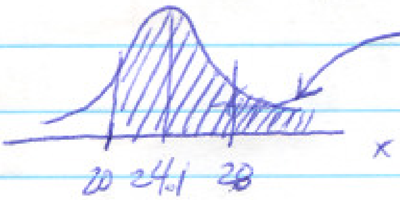
$$a = -7.6918, \quad b = .8309$$

e. z-score for 12 = 2.2786, which means that the ~~Actual 12 is~~ length-to-width ratio of 12 is 2.2768 standard deviations above the mean of all 41 ratios.

#8. Let X = max oxygen uptake (ml/kg) for a patient who regularly exercises

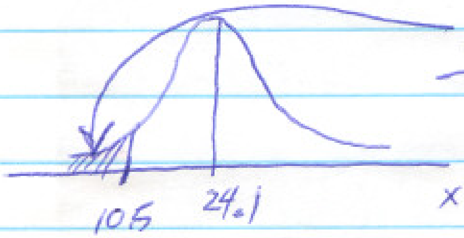
$X \sim N(\mu = 24.1, \sigma = 6.3)$

(a)



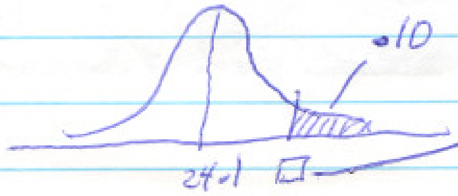
$1 - .2576 = \boxed{.7424}$ (1- cumulative)

(b)



$\boxed{.0154}$ (cumulative)

(c)

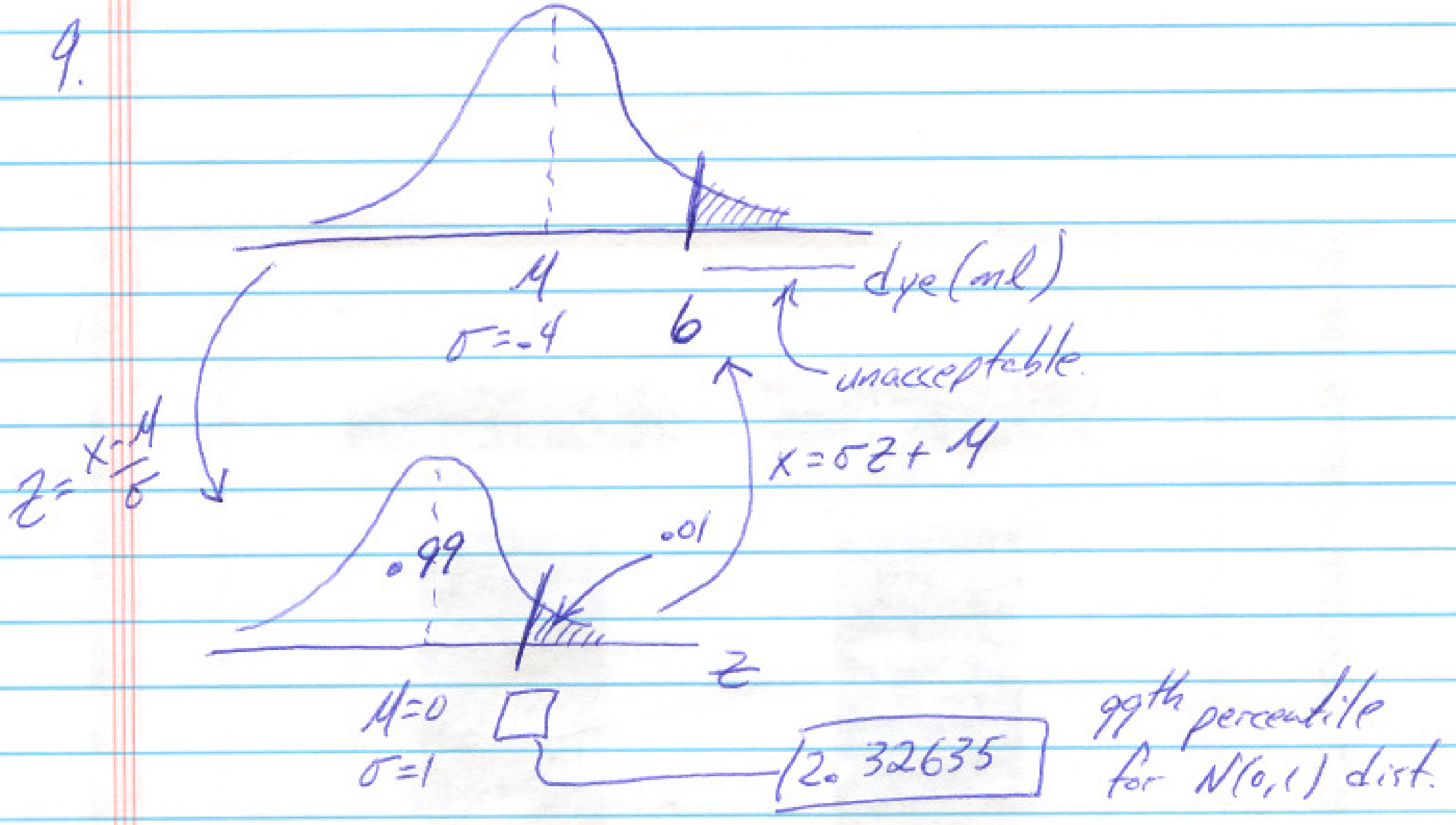


$\boxed{32.1738}$ invcdf

$(32.1738, \infty)$

Elements of Statistics Final Exam – Fall 2004.

9.



$$6 = .4(2.32635) + \mu$$

$$\Rightarrow \quad \mu = 6 - .4(2.32635)$$

$$\boxed{\mu = 6 - .9305 = 5.0695}$$